Context:

Replenishment at Sea (RAS) is a procedure that allows the transfer of fuel, goods, and personnel from one ship to another while they are under way.

In essence it is the same as establishing a flying fox between two points. However, unlike on land, it is impossible just to walk from one point to another.

To establish a physical link between the ships, a gunline (very thin line) is fired first. This process involves mounting a plastic projectile (called a dongle) onto which the gunline is secured, to the end of the barrel of a rifle.

The operator sending the dongle loads the gun with a blank cartridge, aims at the receiving ship, and fires at an angle in such a way that the dongle reaches the other ship. Once the slender gunline has been secured by the receiving ship, stronger and stronger lines are being paid out by the sending ship and hauled in by the receiving ship until the Jackstay (very strong line) can be secured between the two ships. Then, secondary lines (telephone, inhaul and outhaul lines) are positioned.

Have a look at this video to have a better idea of the whole process:

http://www.youtube.com/watch?v=sAlATn4xm4I

Objective:

From a Physics point of view of course, it is possible to do calculations relating to the motion and trajectory of projectiles in the Earth’s gravitational field, which is the object of this work.
First Case Scenario

It is assumed that the decks of the two ships are at equal distance from the surface of the sea. Assume as well for all subsequent calculations that air friction can be ignored.

Mathematical Tools:

- Motion is accelerated
- Acceleration is constant, and downward
- $a = g = 9.81 \text{m/s}^2$
- The horizontal (x) component of velocity is constant
- The horizontal and vertical motions are independent of each other, but they have a common time

Initial position: $x = 0, y = 0$

Initial velocity: $v_i = v_i [\theta]$

Velocity components:
- $x$ direction: $v_{ix} = v_i \cos \theta$
- $y$ direction: $v_{iy} = v_i \sin \theta$

**Equations of motion:**

<table>
<thead>
<tr>
<th></th>
<th><strong>X</strong></th>
<th><strong>Y</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Uniform motion</td>
<td></td>
<td>Accelerated motion</td>
</tr>
<tr>
<td><strong>ACCELERATION</strong></td>
<td>$a_x = 0$</td>
<td>$a_y = g = -9.81 \text{ m/s}^2$</td>
</tr>
<tr>
<td><strong>VELOCITY</strong></td>
<td>$v_x = v_i \cos \theta$</td>
<td>$v_y = v_i \sin \theta + g t$</td>
</tr>
<tr>
<td><strong>DISPLACEMENT</strong></td>
<td>$x = v_i t \cos \theta$</td>
<td>$y = v_i t \sin \theta + \frac{1}{2} g t^2$</td>
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**Initial Conditions:**

Distance between firing point (sending ship) and point of impact (receiving ship) : 55 m

Initial velocity of the dongle : 30 m/s

**ACTIVITIES**

**QUESTION 1 : Flight time**

By using the following formula, find an expression for the total flight time $\Delta t$ as a function of the initial velocity $V_i$ and the initial angle $\theta$.

( Remember that after a span of time $\Delta t$, the altitude $y$ will be zero again )

$$y = v_i t \sin \theta + \frac{1}{2} g t^2$$
QUESTION 2: Range

By using the formula for $x$ as a function of the time $t$ and the angle $\theta$, as well as the formula you found in Question 1, find an expression of the range $\Delta x$ (which is the total horizontal distance travelled during the span of time $\Delta t$), as a function of the initial velocity $V_i$ and the initial angle $\theta$, that is to say a formula independent from the travel time $t$.

\[
x = v_i t \cos \theta
\]

\[
\Delta t = \frac{2 v_i \sin \theta}{-g}
\] (From Question 1)
QUESTION 3 (Application of Question 2): Firing angle

Now that we have an expression for the range as a function of the initial velocity $V_i$ and the firing angle $\theta$, calculate the minimal angle needed to reach the other ship if $V_i = 60 \text{ m/s}$ and the range $\Delta x = 55 \text{ m}$. 
QUESTION 4: Firing angle - extension

Investigate the possibility of firing at a different angle to obtain the same range.
Application: calculate the value of the second angle that would give a range of 55 m.

Hint: \[ \sin (2(90 - \alpha)) = \sin (180 - 2\alpha) = \sin (2\alpha) \]
QUESTION 5: Firing angle - extension 2

Explain why it is preferable to fire the dongle at the smallest of the two angles. Think about conditions at sea.