The aim of this document is to give students the opportunity to trace at first simple parabolas, then more complex ones and use the determinant to find the zeros of a quadratic function.
**PART 1**

*Study the simplest of all Quadratic functions: \( f(x) = y = x^2 \)*

**Activity 1)** Study the function \( x \mapsto f(x) = y = x^2 \)

by calculating \( y \) for the following different values of \( x \) and by completing the table below.

If \( x = -6 \) then \( y = \)  
If \( x = -5 \) then \( y = \)  
If \( x = -4 \) then \( y = \)  
If \( x = -3 \) then \( y = \)  
If \( x = -2 \) then \( y = \)  
If \( x = -1 \) then \( y = \)  
If \( x = 0 \) then \( y = \)  
If \( x = 1 \) then \( y = \)  
If \( x = 2 \) then \( y = \)  
If \( x = 3 \) then \( y = \)  
If \( x = 4 \) then \( y = \)  
If \( x = 5 \) then \( y = \)  

**Activity 2)** Summarise your results by completing the table below.

<table>
<thead>
<tr>
<th></th>
<th>-6</th>
<th>-5</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
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<td>( y )</td>
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</tbody>
</table>
ACTIVITY 3) Trace

\[ y = f(x) \]

The graph you have just traced is called a parabola and is the shape used to design satellite dishes all around the world. What you just traced doesn't look like a satellite dish? Go to part 2 and see.
PART 2

Study this Quadratic function: \( f(x) = y = 0.1 \, x^2 \)

**Activity 1)** Study the function \( x \leftrightarrow f(x) = y = 0.1 \, x^2 \)

by calculating \( y \) for the following different values of \( x \) and by completing the table below.

If \( x = -7 \) then \( y = \) 
If \( x = -6 \) then \( y = \) 
If \( x = -5 \) then \( y = \) 
If \( x = -4 \) then \( y = \) 
If \( x = -3 \) then \( y = \) 
If \( x = -2 \) then \( y = \) 
If \( x = -1 \) then \( y = \) 
If \( x = 0 \) then \( y = \) 
If \( x = 1 \) then \( y = \) 
If \( x = 2 \) then \( y = \) 
If \( x = 3 \) then \( y = \) 
If \( x = 4 \) then \( y = \) 
If \( x = 5 \) then \( y = \) 
If \( x = 6 \) then \( y = \) 
If \( x = 7 \) then \( y = \) 

**Activity 2)** Summarise your results by completing the table below.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-7</th>
<th>-6</th>
<th>-5</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
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<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
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</tbody>
</table>
The graph you have just traced looks much more like a satellite dish now. You should now perceive that the coefficient that multiplies the square term $x^2$ determines how sharp or blunt the vertex (pointy bit) of the parabola is.
PART 3

Study this Quadratic function: \( f(x) = y = x^2 + 2x - 8 \)

**Activity 1)** Study the function \( x \mapsto f(x) = y = x^2 + 2x - 8 \)

by calculating \( y \) for the following different values of \( x \) and by completing the table below.

If \( x = -7 \) then \( y = \) 
If \( x = -6 \) then \( y = \) 
If \( x = -5 \) then \( y = \) 
If \( x = -4 \) then \( y = \) 
If \( x = -3 \) then \( y = \) 
If \( x = -2 \) then \( y = \) 
If \( x = -1 \) then \( y = \) 
If \( x = 0 \) then \( y = \) 
If \( x = 1 \) then \( y = \) 
If \( x = 2 \) then \( y = \) 
If \( x = 3 \) then \( y = \) 
If \( x = 4 \) then \( y = \) 
If \( x = 5 \) then \( y = \) 
If \( x = 6 \) then \( y = \) 
If \( x = 7 \) then \( y = \) 

**Activity 2)** Summarise your results by completing the table below.

<table>
<thead>
<tr>
<th>( x )</th>
<th>-7</th>
<th>-6</th>
<th>-5</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td></td>
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</tbody>
</table>
The graph you have just traced cuts the X axis on two points. These points are called the zeros of the function. These are the values of $x$ for which $f(x) = y = 0$.

There is a way to find these values without having to trace the graph. This is the object of the next activity (Activity 4).
**Activity 4) USE of the DETERMINANT**

Calculate the value(s) of the zero(s) of this function.

<table>
<thead>
<tr>
<th>Remember: General form of a Quadratic</th>
<th>( y = a X^2 + b X + c )</th>
<th>With ( \Delta = b^2 - 4ac )</th>
</tr>
</thead>
<tbody>
<tr>
<td>If ( \Delta &lt; 0 ) then: No solution (No zeros)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>If ( \Delta = 0 ) then: one zero ( x_0 = \frac{-b}{2a} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>If ( \Delta &gt; 0 ) then: two zeros ( x_{01} = \frac{-b - \sqrt{\Delta}}{2a} ), ( x_{02} = \frac{-b + \sqrt{\Delta}}{2a} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

This is what it means from a graph's point of view

- **No zeros**
  - \( \Delta < 0 \)
  - \( a > 0 \)
  - \( a < 0 \)

- **1 zero**
  - \( \Delta = 0 \)
  - \( a > 0 \)
  - \( a < 0 \)

- **2 zeros**
  - \( \Delta > 0 \)
  - \( a > 0 \)
  - \( a < 0 \)

Use these formulas to first calculate the value of Delta (\( \Delta \)) to determine how many zeros

\[ y = x^2 + 2x - 8 \]

has.

Then, calculate the values of these zeros.

Check your values against the graph