The aim of this document is to give students the opportunity to think about the propagation and transfer of electromagnetic energy, in particular when applied to Radar systems such as those of the Air Warfare Destroyer.
Imagine yourself looking at a lightbulb at night, from 20 centimetres away, 2 metres, 10 metres, 100 metres away.

You probably would find it quite painful to look at the light from 20 cm away, whereas it would seem quite dim from a distance of 100 metres away.

What happened? It is the same lightbulb, your eyes have not changed and yet, the amount of light that you detect changes.

This is due to the fact that the available light spreads in all directions and as you move further and further away from the source, the same amount of light produced every second by the lightbulb is spreading thinner and thinner on spheres centered on the light bulb.

So, you can see that the energy available per surface unit (square metre) decreases as you move away from the source.

This available energy is inversely proportional to the square of the distance R.

For instance, at 4 times the original distance, the energy available per square metre is not 4 times less, it is $16 \left(4^2\right)$ times smaller than the original available energy.
Radar waves behave in the same manner as light waves. We can therefore use the property seen in Part 1.

We will consider the case where the emitter and the receiver for the radar are in the same location. That is to say that the antenna broadcasts a signal and also receives the returning signal that bounced back on a target (say an aircraft for instance).

Let us call the Power Emitted by the Radar from point A, $P_{E(A)}$, and the Power Received in point B, $P_{R(B)}$.

The surface area of the sphere centered on the Radar (Emitter) and of Radius $R$ is equal to $4 \times \pi \times R^2$.

Therefore, we can write (according to Part 1) that:

$$P_{R(B)} = \frac{P_{E(A)}}{4 \pi R^2}$$
Now, when the radar signal hits the aircraft, transfers of energy occur (Year 9 ACARA Science).

Part of the signal is absorbed by the structure of the aircraft (if you are interested, check-out Stealth Technology), and another part is reflected back to the receiving antenna of the Radar.

So, everything happens as if the aircraft itself has become a source of radiating energy. Point B becomes an emitter and Point A becomes the receiver. The signal will have to travel the same distance $R$ but the other way around (light blue waves on the diagram).

In the next calculation we will assume that the aircraft is a perfect reflector to simplify things (a full refined formula will be given later on).

\[
P_R(A) = \frac{P_E(B)}{4 \pi R^2}
\]

but \(P_E(B) = P_R(B)\)

because the aircraft (B) is reflecting the signal that came from the radar (A)

and we know that:

\[
P_R(B) = \frac{P_E(A)}{4 \pi R^2}
\]

Therefore:

\[
P_R(A) = \frac{P_E(B)}{4 \pi R^2} = \frac{P_R(B)}{4 \pi R^2} = \frac{P_E(A)}{4 \pi R^2} = \frac{P_E(A)}{(4 \pi R^2)^2}
\]

This means that the power received back by the radar from the original signal is inversely proportional to the Fourth mathematical power of the radius $R$ (Distance Radar to Target)

\[
P_R(A) = \frac{P_E(A)}{16 \pi^2 R^4}
\]

**Formula (1)**
NOTE: The complete Radar Equation looks like this:

$$P_{R(A)} = \frac{G \sigma A_e}{(4 \pi^2 R^2)^2} P_{E(A)}$$

Formulation (2)

Where:
- \(G\) is the power Gain of the antenna 
  (This accounts for concentration of the beam leaving the Radar)

- \(\sigma\) (sigma) is the cross section of the target 
  (A big target like a jumbo jet will reflect more signal than a small jet fighter for instance)

- \(A_e\) is the antenna effective area 
  (This depends on the size and geometry of the antenna)

However, you can see that this formula still looks like:

$$P_{R(A)} = \frac{k}{R^4} P_{E(A)}$$

Because we can group all the constant factors and call them constant term \(k\).
PART 3 : Applications to calculations relating to Radars.

Exercise #1:

A Radar of emitting power 100 kilo Watt (1 kW = 1000 Watt) can detect a specified target at a maximum range (distance R) of 100 km.

How much power would be needed to be able to detect the same target but at a maximum distance R of 500 km?

Use Formula (1) to answer this question.

Even if we do not know what the value of "k" is, we can still do the calculation because, we can use ratios.

Let us use the index (1) for the first case and the index (2) for the second case scenario.

We then have:

> R1 = 100 km , R2 = 500 km

> PR (2) = PR (1) (because we need the same detection threshold)

We then can write:

\[ P_{R(2)} = P_{R(1)} = \frac{k}{(R2)^4} \]

\[ P_{E(2)} = \frac{k}{(R1)^4} \]

Therefore, \[ P_{E(2)} = \frac{(R2)^4}{(R1)^4} \]

\[ P_{E(1)} = \frac{(5)^4 x (R1)^4}{(R1)^4} \]

\[ P_{E(1)} = 625 x P_{E(1)} \]

What this means is that if you want the range to be multiplied by 5 only, you have to provide 625 times the emitting power to the Radar (Supposing that the electronics can take it). So \[ P_{E(2)} \] needs to be:

Therefore, \[ P_{E(2)} = 625 \times 100\ kW = 62,500\ kW = 62.5\ MW\ (=\ 62,500,000\ W) \]
Once you have done the calculation, take a step back and have a look at the value you found. Do you think that this is reasonable?

To help you answer this question, go on the Internet and research what kind of power in Mega Watt the Pelican Point Power Plant in Adelaide is able to generate (or pick a Power Plant closer to where you live).

\[ 1 \text{ MegaWatt} = 1 \, 000 \, 000 \, W \]

The Pelican Point Power Plant can generated up to 480 MW. It can provide electricity to up to 320,000 average homes.

So, it means that the Radar in the previous question would need to use the electricity that would normally be used by:

\[ \frac{62.5}{480} \times 320,000 = 41667 \text{ homes, just for one Radar !} \]

More information on the Pelican Point Power Station on:

Exercise #2:

A Radar uses electro magnetic waves that travel at the speed of light "c".
("c" stands for Celerity which is a Greek word meaning Speed)

\[ c = 300\,000 \text{ kilometres per second} \]
Just imagine, if light could go around the world instead of traveling in a straight line, it would have time to go around the whole of our planet 7.5 times in one second WOW !!!

Calculate how long it takes for a pulse from the Radar to be emitted and come back to the antenna if the target is 150 km away from the radar.

Explain then why Radars are able to follow the trajectory of objects in real time, even if to a human scale, these objects are very, very fast (such as missiles and jet fighters).

The total distance that the signal has to travel will be:

\[ 150\, \text{km} \times 2 \text{ (There and back)} \]

\[ V = \frac{d}{t} \quad \text{Therefore} \quad t = \frac{d}{V} \quad \text{or in this case} \quad t = \frac{d}{c} \]

\[ t = \frac{300 \text{ km}}{300\,000 \text{ km/s}} = 1 \div 1000 \text{ s} = 0.001 \text{ seconds}! \]

No wonder Radars can track missiles and aircrafts. Even if an aircraft is flying at the speed of sound (about 330 m/s) in 1 thousandth of a second, the plane will have moved only by 330 m/s x 0.001 s = 0.33 metres.