Hobart Class DDG (Air Warfare Destroyer)

Objective:

To calculate the volume of the immersed part of the hull (the part painted in brown in the drawing), that is to say, what is called the displacement of the hull of the AWD.

The shape of the hull of a ship can be approximated as a collection of triangular, rectangular and trapezoidal prisms.
Method

It can therefore be seen that the whole volume can be broken down into three prisms. To find out the displacement of the hull, we will calculate the volume of each of the three sections and add them at the end.

To do this, we only need to know:

The value of:
- The length of the triangular front (Called the Prow)
- The length of the rectangular central part (Called Midships)
- The length of the trapezoidal part (Called the Stern)
- The Beam (the width of the ship)
- The width of the stern (small dimension of the trapezium)
- The Draft (distance from the waterline to the bottom of the keel).

Mathematical Tools:

Volume of a triangular prism:

\[ V = \frac{1}{2} \times w \times B \times h \]

\[ V1 = \frac{1}{2} \times w \times L1 \times h \]
Volume of a rectangular prism:

\[ V = wL \times h \]

Volume of a trapezoidal prism:

\[ V = \left( \frac{1}{2} (w+t) \right) L \times h \]
Application to the AWD:

Length of the triangular front (Called the Prow)
$L_1 = 50 \text{ m}$

Length of the rectangular central part (Called Midships)
$L_2 = 55 \text{ m}$

Length of the trapezoidal part (Called the Stern)
$L_3 = 30 \text{ m}$

Beam (the width of the ship)
$w = 18.6 \text{ m}$

Width of the stern (small dimension of the trapezium)
$t = 14.4 \text{ m}$

Draft (distance from the waterline to the bottom of the keel).
$h = 5.17 \text{ m}$

In the following pages, calculate in cubic metres the values of $V_1$, $V_2$ and $V_3$.
Volume of the prow \( V1 \):

Volume of a triangular prism:

\[ V = \frac{1}{2} \times w \times L \times h \]

\[ V1 = B \times h \]

\[ V1 = (\frac{1}{2} \times w \times L1) \times h \]

\[ L1 = 50 \text{ m} \]
\[ w = 18.6 \text{ m} \]
\[ h = 5.17 \text{ m} \]

Your calculations:

\[ V1 = B \times h \]
\[ V1 = (\frac{1}{2} \times w \times L1) \times h \]
\[ V1 = (\frac{1}{2} \times 18.6 \times 50) \times 5.17 \]
\[ V1 = 2404 \]

\[ V1 = 2404 \text{ m}^3 \]
Volume of a rectangular prism:

\[ V = w \times L \times h \]

\[ V_2 = B \times h \]

\[ V_2 = (w \times L_2) \times h \]

\[ L_2 = 55 \text{ m} \quad w = 18.6 \text{ m} \quad h = 5.17 \text{ m} \]

Your calculations:

\[ V_2 = B \times h \]

\[ V_2 = (w \times L_2) \times h \]

\[ V_2 = (18.6 \times 55) \times 5.17 \]

\[ V_2 = 5289 \]

\[ V_2 = 5289 \text{ m}^3 \]
**VOLUME OF Stern V3:**

Volume of a trapezoidal prism:

\[ V = \left( \frac{1}{2} (w + t) L \right) \times h \]

\[ V3 = B \times h \]

\[ V3 = \left( \frac{1}{2} (w + t) L3 \right) \times h \]

\[ L3 = 30 \text{ m} \quad w = 18.6 \text{ m} \quad t = 14.4 \text{ m} \quad h = 5.17 \text{ m} \]

**Your calculations:**

\[ V3 = B \times h \]

\[ V3 = \left( \frac{1}{2} (w + t) L3 \right) \times h \]

\[ V3 = \left( \frac{1}{2} (18.6 + 14.4) \times 30 \right) \times 5.17 \]

\[ V3 = 2559 \text{ m}^3 \]
Now add the three volumes V1, V2 and V3:

\[ V_{\text{total}} = V1 + V2 + V3 \]
\[ = 2404 + 5289 + 2559 \]
\[ = 10252 \]

\[ V_{\text{total}} = 10252 \text{ m}^3 \]

**DISPLACEMENT:**

The volume you have just calculated is called the displacement of the ship. You found it in cubic meters but it is more practical to transform it into metric tons.

1 cubic meter of water is equivalent to 1 metric ton.

\[
\text{AWD Displacement} = 10252 \text{ tons}
\]

**DISPLACEMENT (Refinement):**

The true value of the actual displacement has to take into account the fact that the bottom of the ship is rounded. Also, you can see on the diagram in page 1 that the hull of the ship is quite hollow just above the propellers.

To account for these parameters, architects use what is called a prismatic coefficient. Here, it is equal to about 0.68.

To use it, simply multiply the raw value you calculated before by 0.68.

\[
\text{True AWD Displacement} = 10252 \times 0.68 = 6971 \text{ tons}
\]

You should find a result close to 7000 t